Review Draft Plan

General Idea

My review plan is motivated by the open research problem proposed by Scott Aaronson in 2015[1]. In this paper, Scott emphasised the crucial assumptions underpinning the quantum machine learning algorithms. In particular, he examined four caveats of the HHL algorithm[2], which lies at the heart of quantum machine learning. Although the author has managed to deliver many sound and inspiring arguments on the rivalry between classical and quantum computing in his paper, my greatest take-away from this paper is more ethical than technical — "...along with the excitement (of the new quantum machine learning algorithms), we ought to maintain a sober understanding of what these algorithms would and wouldn't do: an understanding that the original papers typically convey, but that often gets lost in secondhand accounts." Frankly speaking, as a beginner in the field, I often find myself in the trap of this kind of 'un-sober understanding' — knowing what an algorithm can ideally do, but failing to realise what it cannot practically achieve. Hence, I wish to write a review on the assumptions of HHL algorithm and its affiliated applications in quantum machine learning to hopefully provide a more comprehensive understanding of the "real picture" to beginners including myself.

The review will be divided into three parts (ideally, if time permits): 1) detailed analysis and evaluation of the assumptions in HHL algorithm; 2) brief analysis of the assumptions made when HHL algorithm is being extended to develop other quantum machine learning algorithms; 3) comparison between the original quantum machine learning algorithms and quantum-inspired classical counterparts

Part I: Detailed Examination of HHL Algorithm

HHL algorithm is introduced to 'partially solve' a linear system of equations in the form $Ax = b$ and it is a combination of several basic quantum algorithms, including phase estimation and Hamiltonian simulation. The efficient implementation of each step requires a set of strict assumptions. Besides, in order for HHL algorithm to have exponential speed-up, there is a restriction on the input (what knowledge we must already have on the matrix A and vector b) and output (clearly not x, but some information on x) of the problem. This section aims to examine the HHL algorithm step by step and extract the assumptions in each individual step. To analyse and evaluate the assumptions, the review aims to answer the following specific questions for each relevant assumption:

[1] Scott Aaronson. *Quantum Machine Learning Algorithms: Read the fine print.* 2015

[2] Aram Harrow, Avinatan Hassidim, Seth Lloyd. *Quantum algorithm for linear systems of equations.* 2009

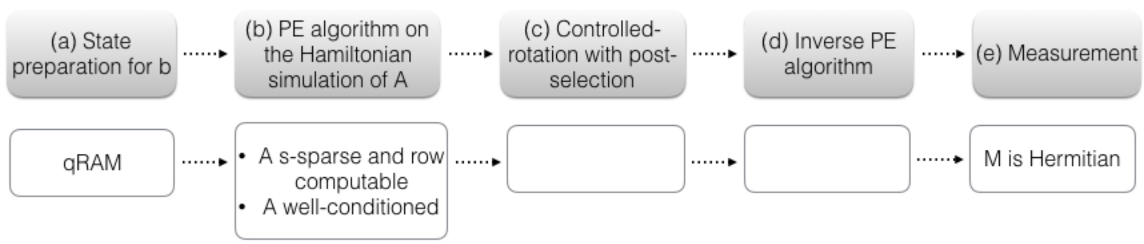
1) (Analysis) What is the objective of introducing this assumption? What would happen to the complexity or/and probability error of the algorithm without this assumption?

2) (Evaluation) How likely does the assumption hold true? Usually, there are two types of assumptions: those which limit the scope of the discussion and those which have the potential to be right.

• For the first type of assumptions, the likelihood of its being true is an issue of frequency — how often will we see a problem in the restricted scenarios where the algorithm works? (aka examine the usefulness of the algorithm despite its assumption).

• For the second type of assumptions, the likelihood of its being true is about feasibility — how likely can we achieve the assumed situation given the physical constraints? (aka propose further research topics to look at).

The examination of each assumption will be centred around these questions, but there might be some other pertinent questions coming into my mind as my reading proceeds. This section will be sub-divided based on the steps in HHL algorithms, as illustrated in the figure below (the first row is the flow of HHL algorithm and the second row is the (incomplete) assumptions for each corresponding step).



The original paper of HHL algorithm includes the detailed analysis of some assumptions and many relevant cross-references. Hence, my review for this section will start from this paper and its cross-references.

Part II: Brief Analysis of Extended Quantum Machine Learning Algorithms

High-dimensional linear algebra lies at the heart of most machine learning algorithms. In a public lecture given by Seth Lloyd[3], he mentioned that common methods in machine learning including implementing Fast Fourier Transform, finding eigenvalues/eigenvectors and performing matrix inversion. With only a brief understanding of basic machine learning algorithms myself, I could roughly see why the last two tasks are relevant, but not so sure about the FFT for now. And in fact, some very important machine learning algorithms do not require anything as complicated as FFT or matrix inversion; the main task is just doing inner product and finding the distance.

[3] $https://www.youtube.com/watch?v = Lbndu5EIWvI$

In this section, the review aims to look at the existing quantum machine learning algorithms and some of them are listed below. As suggested by Scott in his *Fine Print*, many quantum machine learning algorithms just give a 'general template' instead of a feasible tool. Hence, to briefly analyse each algorithm, the review aims to answer the following questions by summarising the existing literature:

1) What is the underlying linear algebra question behind this machine learning problem and what is the role of quantum algorithm in solving this question?

2) What are the underlying assumptions behind the quantum template and how confident are we about those assumptions? If one or some of the assumptions are unlikely to achieve in practical sense, does this quantum machine learning algorithm still have advantage over its classical counterpart?

1307.0411	Quantum algorithms for supervised and unsupervised machine learning	Seth Lloyd, Masoud Moseni, Patrick Rebentrost
1204.5242	Quantum Data-Fitting	Nathan Wiebe, Daniel Braun, Seth Lloyd
1307.0471	Quantum support vector machine for big data classification	Patrick Rebentrost, Masoud Mohseni, Seth Lloyd
1307.0401	Quantum principal component analysis	Seth Lloyd, Masoud Moseni, Patrick Rebentrost
1603.08675	Quantum Recommendation Systems	Iordanis Kerenidis, Anupam Prakash

In fact, the second question naturally leads to Part III of this review and the best I could do is doing a summary of the existing arguments, probably paper-wise, rather than topic-wise.

Part III: Comparison with Quantum-inspired Classical Algorithms

To be honest, without having done the first two parts, I am not sure how this section is going to be structured. My primitive understanding of the existing quantum-inspired classical algorithm is to draw a parallel between qRAM and classical search-and-query process. The argument goes something like qRAM and search-and-query are on some sort of equal ground. And if we can achieve the search-and-query process classically, then many quantum machine learning algorithms don't have the exponential speed-up they claim to be. Hence, for this section, I will just leave a list of possible papers to look at first and leave the detailed plan a bit later.

1811.04909	Quantum-inspired low-rank stochastic regression with logarithmic dependence on the dimension	Andras Gilyen, Seth Lloyd, Ewin Tang
1811.00414	Quantum-inspired classical algorithms for principal component analysis and supervised clustering	Ewin Tang
1807.04271	A quantum-inspired classical algorithm for recommendation systems	Ewin Tang